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THE RADIATION OF NON-STATIONARY SPHERICAL PRESSURE WAVES BY A MOVING AND PARTIALLY PERMEABLE BOUNDARY†

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The radiation (generation) of pressure waves by a spherical cavity is investigated using the non-linear time-transformation method in wave initial-boundary-value problems with specified Neumann-type boundary conditions on a moving and partially permeable boundary [1, 2]. The results obtained reflect the hydrodynamic processes which accompany underwater explosions of different physical kinds and of limited power.

1. PHYSICAL PREREQUISITES AND THE MATHEMATICAL FORMULATION OF THE PROBLEM

When an energy pulse is liberated in a limited volume of liquid, vapour-gas or plasma cavities are formed, often of spherical form, and pressure waves are generated when these expand (collapse). The dynamics of a cavity are determined both by the physical processes in the cavity and by the parameters of the liquid medium. The kinematic characteristics of the cavity (the initial value of the radius, the variation of the radius with time and the inflow of liquid) are unknown in advance and must be determined by solving the overall problem, which includes both the internal physical problem and the external hydrodynamic one. However, the overall problem can only be solved numerically. The external problem can, however, be solved analytically, which is of independent interest. Thus, the determination of the external pressure field in the liquid from the results of optimal measurements of the radius of the channel over time is the basis for diagnostics of the pressure in the plasma of a high-voltage electric discharge in a liquid [3].

Confining ourselves to small values of the velocity of the liquid with respect to the velocity of sound in it, and simultaneously bearing in mind the finite nature of the amplitude of the displacement of the cavity boundary, we will describe the dynamics of the perturbed motion of the liquid by a linear wave equation, while the Neumann boundary condition will be specified on the moving boundary and its instantaneous position. The correctness of this approach and its area of applicability were discussed previously in [2, 4, 5]. Hence, assuming the motion of the liquid to be potential, we can write the initial-boundary-value problem in the form

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}-\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2}=0 \tag{1.1}$$

$$r=R(t):\ \frac{\partial\varphi}{\partial r}=\upsilon_s(t) \tag{1.2}$$

$$t=0:\ \varphi=\frac{\partial\varphi}{\partial t}=0,\quad R=R_0 \tag{1.3}$$

where φ is the potential of the velocities of perturbed motion of the liquid, r is the coordinate, t is the time, c_0 is the velocity of sound, $R=R(t)$ is the variation of the radius of the cavity with time and υ_s is the velocity of the particles of liquid on the contact boundary of the cavity.

From the known potential of the velocities, the velocity field and the pressure field in the linear approximation are given by the expressions

$$\upsilon(r,t)=\frac{\partial\varphi}{\partial r},\quad p(r,t)=-\rho_0\frac{\partial\varphi}{\partial t} \tag{1.4}$$

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We will consider the kinematic boundary condition (1.2) in more detail. In the case of an infinitesimal pulsation amplitude of the cavity, the boundary condition is specified on the initial position of the boundary [6, 7], as was done for the non-stationary problem [6] and for the case of the radiation of harmonic waves [7]. When the boundary of the cavity is impermeable, both in the case of a small amplitude [6, 7] and in the case of the motion of the boundary [8], the velocity of the liquid on the contact boundary is equal to the velocity of motion of the cavity wall, i.e. $\upsilon_s(t) = dR/dt$. In the general case when the inflated envelope is penetrable [9], or there is heat and mass transfer through the cavity wall [10, 11], the radial component of the velocity of the liquid is not equal to the radial velocity of motion of the contact boundary. Thus [10, 11]

$$\upsilon_s(t) = \frac{dR}{dt} - \frac{q}{\tau\rho_0} \tag{1.5}$$

where q is the thermal power and τ is the specific heat of vaporization of the surrounding liquid. When the cavity pulsates in the superheated liquid [10], heat is transferred by conduction from the liquid, i.e. $q = \lambda dT/dr$ when $r = R(t)$, where λ is the thermal conductivity. As the plasma cavity expands [11] the heat is transferred by radiation of hot plasma, i.e. $q = \sigma T^4$, where σ is the Stefan–Boltzmann constant and T is the plasma temperature.

The problem is completely defined in the numerical solution. In order to obtain a unique analytic solution it is also necessary to specify the radiation condition [12–14]. The radiation condition for harmonic waves enables us to distinguish from all the solutions of Eq. (1.1) [12] those which correspond to expanding spherical waves. This condition is trivial for a symmetrical form of the wave solution. In the case of non-harmonic waves, the radiation condition is transformed [13, 14] into the causality principle, the essence of which is that all functions of the wave field, which depend on time (including in complex form), must vanish for a negative value of the wave argument. The analogue of the radiation condition for the problem of a high-power underwater explosion is the condition on the shock-wave front [15].

2. ANALYTIC SOLUTION OF THE INITIAL-BOUNDARY-VALUE PROBLEM WITH MOVABLE BOUNDARIES IN THE GENERAL CASE

We will seek a solution of the initial-boundary-value problem with movable boundaries (1.1)–(1.3), where, in the general case, by (1.5), the radial velocity of the liquid on the boundary is not equal to the rate of change of the sphere radius, using the method of non-linear time conversion [1, 2]. We write the solution of wave equation (1.1), which satisfies initial conditions (1.3) and the radiation condition

$$\varphi(r,t) = f(t^0)/r, \quad t^0 = t - (r - R_0)/c_0 \geqslant 0 \tag{2.1}$$

where t^0 is the wave argument and f is a so-far unknown function. When solution (2.1) satisfies boundary condition (1.2), we obtain the representation for the function f in the form

$$f(\xi) = -c_0 E_w(\xi)\left[C + \int_0^{\xi} \frac{\upsilon_s(w(\xi_1))R(w(\xi_1))}{E_w(\xi_1)} d\xi_1\right] \tag{2.2}$$

$$E_w(\xi) = \exp\left[-c_0 \int_0^{\xi} \frac{d\xi_2}{R(w(\xi_2))}\right]$$

where ξ is a variable which has the meaning of "new time", which is introduced by means of the transformation

$$t - (R(t) - R_0)/c_0 = \xi \tag{2.3}$$

while $t = w(\xi)$ is the inversion of the function $\xi(t)$. We will assume the constant of integration C in (2.2), corresponding to the zero initial conditions (1.3), to be equal to zero. Note that, when the condition $((R(t) - R_0)/(c_0 t))^2 \ll 1$ is satisfied for any form of the function $R(t)$, which has no discontinuities, the inversion $t = w(\xi)$ is possible and unique. Now, using representations (2.1) and (2.2) we obtain, after reduction, the required velocity potential in the form

$$\varphi(r,t) = -c_0 \frac{R(w(t^0))}{r} \Phi(w(t^0)), \quad \Phi(w(t^0)) = E(w(t^0)) \int_0^{w(t^0)} \frac{\upsilon_s(t)}{E(t)} \left(1 - \frac{1}{c_0}\frac{dR}{dt}\right) dt \tag{2.4}$$

$$E(t) = \exp\left[-c_0 \int_0^{t} \frac{dt}{R(t)}\right]$$

For an impermeable boundary when $\upsilon_s(t) = dR/dt$, solution (2.4) becomes the solution obtained previously [2]. When the displacement of the impermeable boundary has a small amplitude $((R - R_0)/R_0)^2 = (\upsilon_s/c_0)^2 \ll 1$, we must take $R = R_0$, $w(t^0) = t^0$ in (2.4). We then obtain Landau's solution [6].

Using the representation for the velocity potential (2.4), by the second formula of (1.4) we obtain the pressure function at a point of the wave zone

$$\bar{p}(r,t) = \frac{\rho_0 c_0^2}{r}\left[R(w(t^0))\frac{\upsilon_s(w(t^0))}{c_0} - \Phi(w(t^0))\right] \tag{2.5}$$

Taking $r = R(t)$ and $r = R_0 + c_0 t$ successively in (2.6), we obtain the following representations for the pressure on the cavity wall and the pressure on the wave front, respectively

$$\bar{p}_s(t) = \bar{\upsilon}_s(t) - \frac{\Phi(t)}{R(t)}, \quad \bar{p}_c(t) = \frac{\bar{\upsilon}_s(+0)}{1+\bar{t}}, \tag{2.6}$$

$$\bar{p}_s = \frac{p_s}{\rho_0 c_0^2}, \quad \bar{p}_c = \frac{p_c}{\rho_0 c_0^2}, \quad \bar{\upsilon}_s = \frac{\upsilon_s}{c_0}, \quad \bar{t} = \frac{c_0 t}{R_0}$$

It is of independent interest to obtain an equation relating the pressure at a point in the wave zone and the pressure on the cavity wall.

Using the second formula of (1.4) we can write the following expressions for the pressures

$$p(r,t) = -\frac{\rho_0}{r}f_1(t), \quad p_s(t) = -\frac{\rho_0}{R(t)}f_1\left(t - \frac{R(t)-R_0}{c_0}\right) \tag{2.7}$$

where $f_1(t) = df/dt$. The required relation between the pressure functions follows from (2.7), namely

$$p_s(t) = \frac{r}{R(t)}p\left(t - \frac{R(t)-R_0}{c_0}\right) \tag{2.8}$$

3. MOTION OF THE BOUNDARY WITH CONSTANT VELOCITY

We will assume that, at the instant of time $t = 0$, the partially permeable cavity wall of radius R_0 begins to move with constant velocity, i.e. $R(t) = R_0 + \upsilon_0 t$. We will also assume that the radial velocity of the liquid on the movable contact boundary is also constant but $\upsilon_1 \leqslant \upsilon_0$. Transformation (2.3) then takes the form $t(1 - M_0) = \xi$, where $M_0 = \upsilon_0/c_0$, whence it follows that

$$R(w(t^0)) = R_0\psi(t^0), \quad \psi(t^0) = 1 + \upsilon_0 t^0/[R_0(1-M_0)] \tag{3.1}$$

and the expression for the velocity potential (2.4) takes the form

$$\varphi(r,t) = -c_0 M_1 \frac{R_0^2}{r}\psi^{-\alpha_-}\frac{\alpha_-}{\alpha_+}(\psi^{\alpha_+} - 1); \quad \alpha_\pm = \frac{1}{M_0} \pm 1, \quad M_1 = \frac{\upsilon_1}{c_0} \tag{3.2}$$

We obtain the following expressions for the velocity and pressure fields

$$\bar{\upsilon}(r,t) = M_1\frac{\alpha_-}{\alpha_+}\frac{R_0}{r}\psi\left[\frac{R_0}{r}\psi(1-\psi^{-\alpha_+}) + \frac{2}{\alpha_-}\left(1 + \frac{\alpha_-}{2}\psi^{-\alpha_+}\right)\right] \tag{3.3}$$

$$\bar{p}(r,t) = 2M_1\frac{R_0}{r}\alpha_+^{-1}\psi\left(1 + \frac{\alpha_-}{2}\psi^{-\alpha_+}\right) \tag{3.4}$$

Taking $r = R_0 + \upsilon_0 t$ in (3.3) we obtain the identity $\bar{\upsilon}_s(t) = M_1$. Taking $r = R_0 + \upsilon_0 t$, from expression (3.4) we obtain a representation for the pressure on the cavity wall

$$\bar{p}_s(t) = \frac{2M_1}{\alpha_+}\left[1 + \frac{\alpha_-}{2}(1 + M_0\bar{t})^{-\alpha_+}\right], \quad \bar{t} = \frac{c_0 t}{R_0} \tag{3.5}$$

For an impermeable boundary, when $\upsilon_0 = \upsilon_1$ and $M_0 = M_1$, solution (3.2) and (3.4) becomes the solution obtained earlier [2]. Assuming $M_1 = M_0$ and $R_0 = 0$ in (3.2)–(3.5) we arrive at the well-known Taylor solution [8].

Further, taking $r = R_0 + c_0t$ in (3.4), we obtain the following representation for the pressure on the wave front

$$\bar{p}_c(t) = M_1/(1+\bar{t}), \quad \bar{t} = c_0t/R_0 \tag{3.6}$$

We now obtain the asymptotic values of (3.5) for small and large values of the time

$$\bar{p}_s(0) = M_1, \quad \bar{p}_s(\infty) = 2M_1/\alpha_+ \tag{3.7}$$

Note that the solution for the non-zero value of the initial radius of the cavity (3.5) reduces to the solution for the zeroth initial radius for large values of the time [8]. This transition is related to the idea of the time τ of the transient [2], found from the equation

$$[\bar{p}_s(0) - \bar{p}_s(\infty)]/[\bar{p}_s(\tau) - \bar{p}_s(\infty)] = e \tag{3.8}$$

where $\bar{p}_s(\tau)$ is the value of the pressure function (3.5) when $t = \tau$, while $\bar{p}_s(0)$, $\bar{p}_s(\infty)$ are found from (3.8). From (3.8) we obtain

$$\tau = \frac{R_0}{c_0M_0}(e^{1/\alpha_+} - 1), \quad \tau \approx \frac{R_0}{c_0(1+M_0)}, \quad M_0 \to 0 \tag{3.9}$$

The second method of estimating the time taken for the system to reach the self-similar state involves specifying the relative error of the pressure δ at the instant of time τ with respect to the value of the pressure (3.7) for large values of the time

$$[\bar{p}_s(\tau) - \bar{p}_s(\infty)]/\bar{p}_s(\infty) = \delta \tag{3.10}$$

Solving (3.10) for τ and taking (3.7) into account we obtain

$$\tau = \frac{R_0}{c_0M_0}\left[\left(\frac{\alpha_-}{2\delta}\right)^{1/\alpha_+} - 1\right] \tag{3.11}$$

If in (3.5) we take the quantities M_0 and M_1 with negative signs, we obtain a representation for the pressure on the cavity wall when it collapses at a constant rate in the form $R(t) = R_0 - \upsilon_0t$. The pressure on the cavity wall, being negative ($\bar{p}_s(0) = -M_1$) when $t = 0$, increases with time and becomes zero when

$$t = \frac{R_0}{c_0M_0}\left[1 - \left(\frac{2}{\alpha_+}\right)^{1/\alpha_+}\right] \tag{3.12}$$

When the time increases further, the pressure becomes a positive quantity and, when $R = 0$, it takes the value $\bar{p}_s = 2M_1/\alpha$.

In Figs 1 and 2 we show the results of a calculation of the pressure on the wall of an impermeable cavity when the radius increases at a constant rate. In Fig. 1 we show a graph of the dimensionless pressure against the dimensionless time $t = c_0t/R_0$ for different numbers M_0 for the same value of the initial radius R_0. In Fig. 2 we show graphs of the pressure for the same value of the rate of expansion of the cavity but different values of the initial radius R_0. The dashed curves correspond to the pressure on the wave front.

In Fig. 3 we show the results of a calculation of the pressure on the wall when a cavity of initial radius R_0 collapses for different values of M_0. The dashed curve corresponds to the instant $R(t) = 0$.

4. PERIODIC COLLAPSE OF A CAVITY

Periodic collapse of a cavity occurs during an underwater explosion. The cavity usually contains three-four decaying pulses, generating the corresponding number of pressure pulses. Electrical explosions in water, as experiment shows, can give a form of the pressure function such that the second pulse has a greater amplitude than the first. The single collapse of a bubble was investigated in [17, 18]. A more

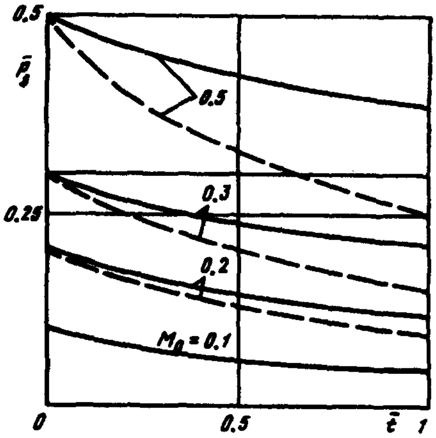

Fig. 1.

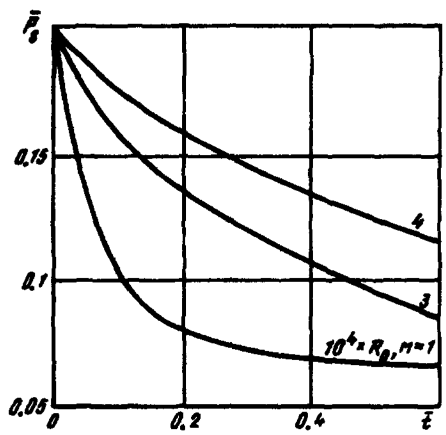

Fig. 2.

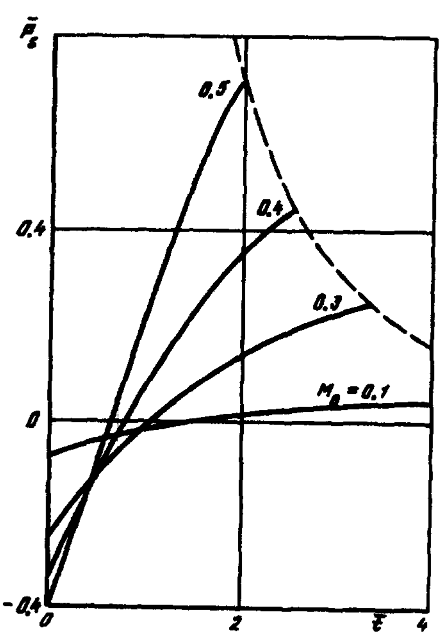

Fig. 3.

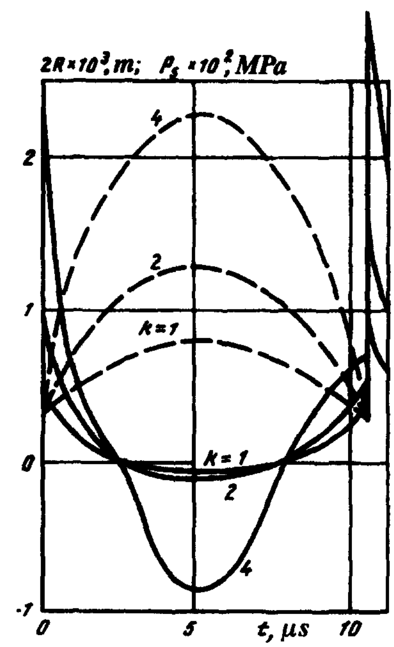

Fig. 4.

complex problem is the problem of the generation of pressure waves by a spherical cavity which pulsates with periodic collapse [19].

Following the approach used previously [19], we will consider the external hydrodynamic problem, assuming pulsations in the form

$$R(t) = R_0 + R_1 \,|\sin \omega_0 t\,| \tag{4.1}$$

where R_0 is the cavity radius at the instant of time $t = 0$, R_1 is the pulsation amplitude and ω_0 is the pulsation frequency, which is related to the pulsation period τ_0 by the relation $\omega_0 = \pi/\tau_0$. Using (4.1) we obtain the following expression for the velocity of motion of the cavity wall

$$v_s(t) = v_0 \cos\omega_0(t - i\tau_0), \quad i = 0, 1, 2, \ldots, \quad v_0 = R_0\omega_0 \tag{4.2}$$

Note that the rate of expansion of the cavity at the initial stage and the rate of collapse at the final stage are equal to $\pm v_0$, respectively. The formal solution of the problem is described by (2.6), where the kinematics of the cavity is given by expressions (4.1) and (4.2).

We will estimate the amplitude of the first and second pressure pulses on the cavity wall. At the beginning of the cavity expansion, according to (4.1) and (4.2), the motion of the wall occurs with constant velocity υ_0. The amplitude of the first pressure on the wall, by (3.7), is equal to M_0. The amplitude of the first pressure pulse on the wall, by (3.7), is equal to M_0. The amplitude of the second pulse consists of two components: $\bar{p}_2 = \bar{p}_{21} + \bar{p}_{22}$, where $\bar{p}_{21}$ is the value of the pressure, equal to the pressure pulse from the second expansion of the cavity and, for a loss-free process, is equal to $\bar{p}_1$, and $\bar{p}_{22}$ is the pressure at the end of the first cavity collapse. By (3.5) we have $\bar{p}_{22} = 2M_0^2/(1 - M_0)$. Substituting the values of the components of the pressure into the expression for the amplitude and the second pulse, we obtain

$$\bar{p}_2 = M_0(1 + M_0)/(1 - M_0) > \bar{p}_1 \qquad (4.3)$$

Hence, in the case of a periodic collapse of the cavity without losses, as given by (4.1), the amplitude of the second pulse is always greater than that of the first. If the losses are taken into account, no unique solution can be reached.

In Fig. 4 we show the results of a numerical calculation of the pressure acting on the wall of an impermeable cavity, as given by (2.7), for three forms of cavity pulsation

$$R(t) = R_0 + kR_1 |\sin \omega_0 t| \qquad (4.4)$$

where $k = 1, 2, 4$ and the following values are assumed: $R_0 = 0.275 \times 10^{-3}$ m, $R_0 = 0.125 \times 10^{-3}$ m and $\omega_0 = 0.3 \times 10^6$ s^{-1}. The dashed curves correspond to the variation of the cavity radius given by (4.4). The continuous curves describe the behaviour of the pressure function on the cavity wall.

5. SUPERPOSITION OF SMALL-AMPLITUDE PULSATIONS ON THE LINEAR GROWTH RATE OF THE SPHERE RADIUS

The problem of the generation of pressure waves when pulsations are superimposed on the linear growth rate of the cavity radius arises when considering the rising from a great depth of a pulsating cavity which has been formed as a result of an underwater explosion. This case is also characteristic for a high-voltage electric discharge in water, for a controlled rate of energy input into the channel in the form of a sequence of pulses [20].

Assuming that the variation of the cavity radius with time is known [20], we can write it in the form

$$R(t) = R_0 + \upsilon_0 t + R_1 f(\omega_0 t), \quad (R_1 / R)^2 \ll 1 \qquad (5.1)$$

where R_0 is the cavity radius at the instant of time $t = 0$, υ_0 is the linear growth rate of the cavity radius, R_1 is the amplitude of small pulsations, f is a certain periodic function, describing the pulsations, and

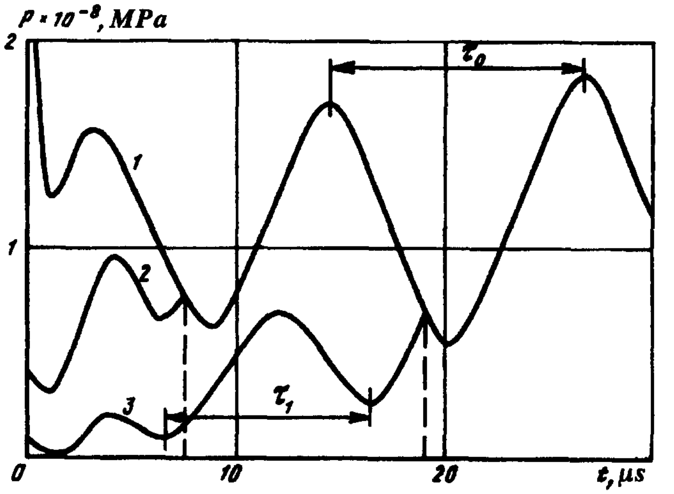

Fig. 5.

ω_0 is the pulsation frequency. From (5.1) we obtain the following representation for the velocity of motion of the cavity boundary

$$v_s(t) = v_0 + v_1(t), \quad v_1(t) = v_1 f_1(\omega_0 t) \tag{5.2}$$

$$v_1 = R_1\omega_0, \quad f_1 = df/dt$$

where υ_1 is the amplitude of the rate of pulsations.

Taking into account the smallness of the pulsation amplitude and the fact that the velocities υ_0 and υ_1 are comparable with one another in the boundary condition of problem (1.2), we can assume

$$R(t) \approx R_0 + v_0 t, \quad v_s(t) = v_0 + v_1 f_1(t) \tag{5.3}$$

Substituting (5.3) into the representation for the pressure on the cavity wall (2.6), we obtain

$$\bar{p}_s = \bar{p}_{s1}(t) + \bar{p}_{s2}(t) \tag{5.4}$$

$$\bar{p}_{s1} = \frac{2}{\alpha_+}\left[1 + \frac{\alpha_-}{2}\left(1 + \frac{v_0 t}{R_0}\right)^{-\alpha_+}\right] \tag{5.5}$$

$$\bar{p}_{s2} = M_1\left[f_1(t) - \frac{\alpha_- v_0}{2R_0}\left(1 + \frac{v_0 t}{R_0}\right)^{-\alpha_+} \int_0^t f_1(\tau)\left(1 + \frac{v_0 \tau}{R_0}\right)^{1/M_0} d\tau\right] \tag{5.6}$$

The first component of the pressure (5.5) in the general sum (5.4) is the result of the linear increase in the cavity radius. This is a continuous function of time. As shown previously, it is close to exponential, and for large values of time takes a constant value $2M_0^2/(1 + M_0)$. The second component of the pressure (5.6) is determined by the nature of the pulsations, but it also depends on the rate of linear expansion, which suggests a breakdown in the superposition principle in problems with moving boundaries.

As an example, we will consider the function which describes the pulsations in the form

$$f(\omega_0 t) = \sin^2 \omega_0 t \tag{5.7}$$

We obtain the pressure function at a point in the wave zone from (2.8), which, for the specific case (5.7), takes the form

$$\bar{p}(r,t) = \frac{R_0}{r}\left[1 + M_0(1 + M_0)\frac{c_0 t}{R_0}\right]\bar{p}_s((1 + M_0)t)$$

In Fig. 5 we show the results of a calculation of the pressure function: 1—on the cavity wall, 2—at a point with coordinate $r/R_0 = 5$, and 3—at a point with coordinate $r/R_0 = 25$. The intersection of the curves denotes the arrival of a pressure wave at the given point of the field. The oscillation period of the pressure function on the cavity wall is equal to the cavity pulsation period $\tau_0 = 2\pi/\omega_0$, while the period of the pressure function at a point in the wave zone decreases in accordance with expression (5.8) $\tau_1 = \tau_0/(1 + M_0)$, which indicates the occurrence of the Doppler effect.

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